The QCD phase diagram from the lattice

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ILGTI: TIFR

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Outline

How many flavours

- Flavour symmetries not exact: difference in masses of different flavours breaks symmetry. For example: if $m_u = m_d$ then (within QCD) any two mutually orthogonal linear combinations of u and d are equivalent: symmetry. When $m_u \neq m_d$, symmetry broken: these transformations change energy.
- Since $m_{\pi^0} \simeq m_{\pi^\pm}$, flavour SU(2) is a good approximate symmetry of the hadron world. Flavour SU(3) is not useful without symmetry breaking terms (Gell-Mann and Nishijima).
- If some $m \gg \Lambda_{QCD}$ then that quark is not approximately chiral. In QCD two flavours are light ($m_{u,d} \ll \Lambda_{QCD}$) and one is medium heavy ($m_s \simeq \Lambda_{QCD}$). Recall that $m_\pi = 0.2 m_\rho$ but $m_K = 0.7 m_\rho$. Limit $m_\pi = 0$: chiral symmetry.
- Do we have a two flavour phase diagram or a three flavour phase diagram, or something else?

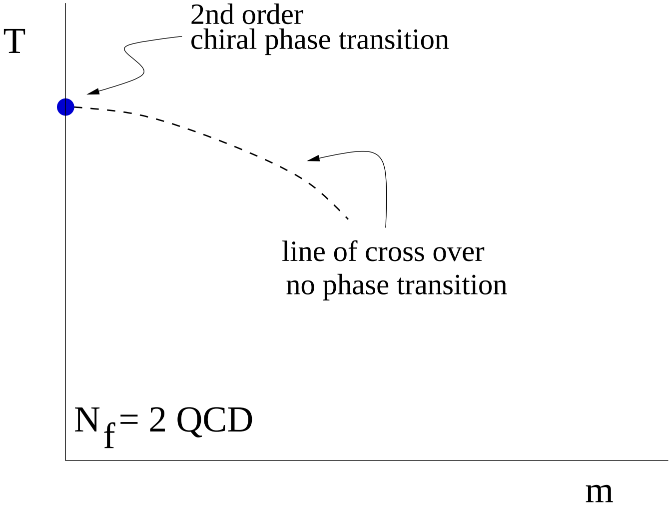
The two flavour world

- What distinguishes the phases? Exact answer only for massless quarks. In the vacuum chiral symmetry is broken; $\langle\overline{\psi}\psi\rangle$ **chiral condensate** is non-vanishing. Pions are the massless fluctuations around the vacuum. At high temperature $\langle\overline{\psi}\psi\rangle = 0$.
- When correlation lengths finite then susceptibility always finite:

$$\chi = \int d^3x C(x), \quad C(x) \simeq \exp(-mx), \quad \xi = 1/m_{PS}.$$

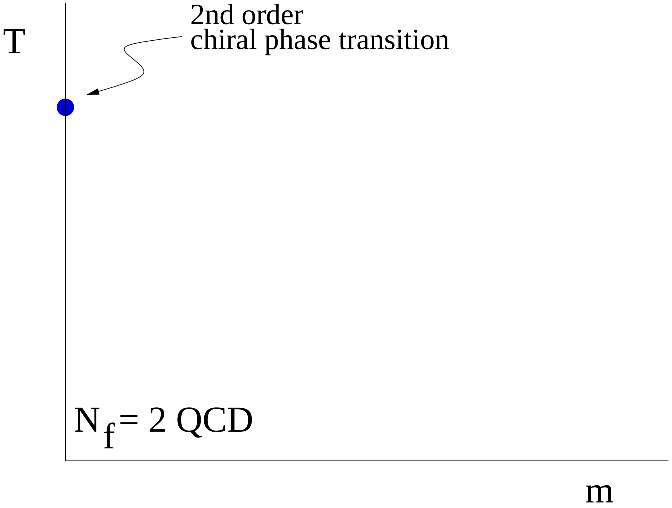
  At critical points correlation lengths diverge; integral diverges, so susceptibility diverges.
- When quarks are massive, then at transition scalar mass degenerate with $m_\pi \neq 0$. No vanishing masses, so all susceptibilities finite. May still have a maximum as T changes: cross over for massive quarks.

The two flavour phase diagram



Pisarski and Wilczek, PR D 29, 338 (1984)

The two flavour phase diagram

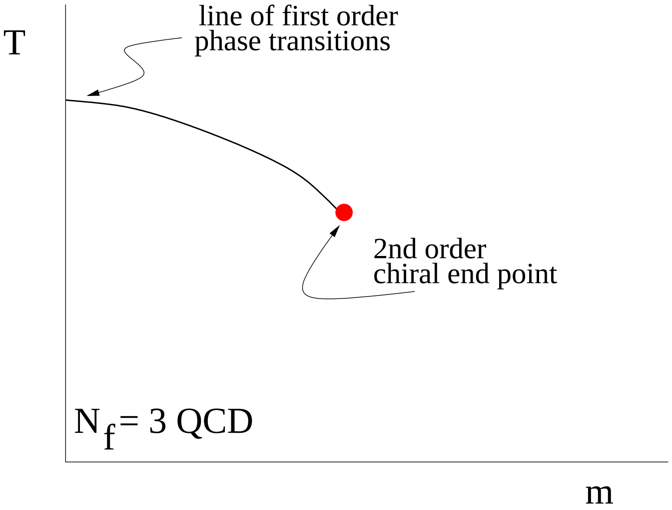


Pisarski and Wilczek, PR D 29, 338 (1984)

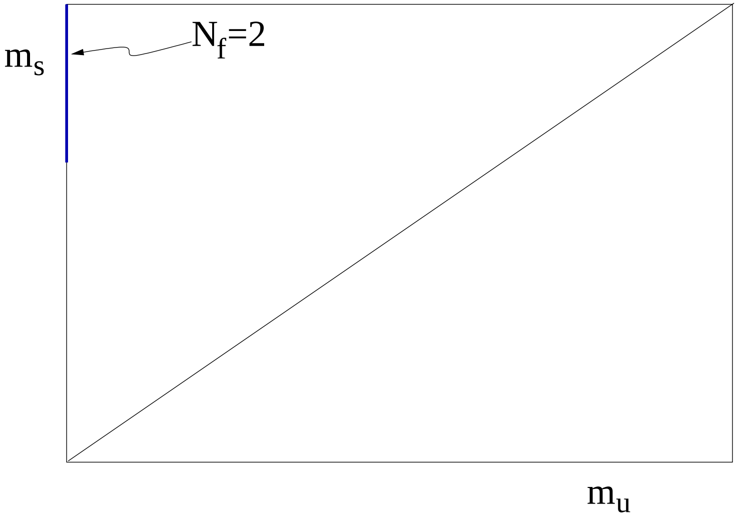
The three flavour world

- For three massless flavours $m_\pi = m_K = m_\eta = 0$. Chiral condensate distinguishes phases. However: first order phase transition; chiral condensate vanishes with a jump.
- If flavour symmetry exact ($m_\pi = m_K = m_\eta \neq 0$), then first order transition remains stable upto some point. Jump decreases continuously until it vanishes. This is a critical end point of this line.
- In the real world SU(3) flavour symmetry is broken ($m_\pi \neq m_K \neq m_\eta$). What is the phase diagram? Encoded in the Columbia plot.

The three flavour phase diagram



The Columbia plot



Brown et al, PRL 65, 2491 (1990)

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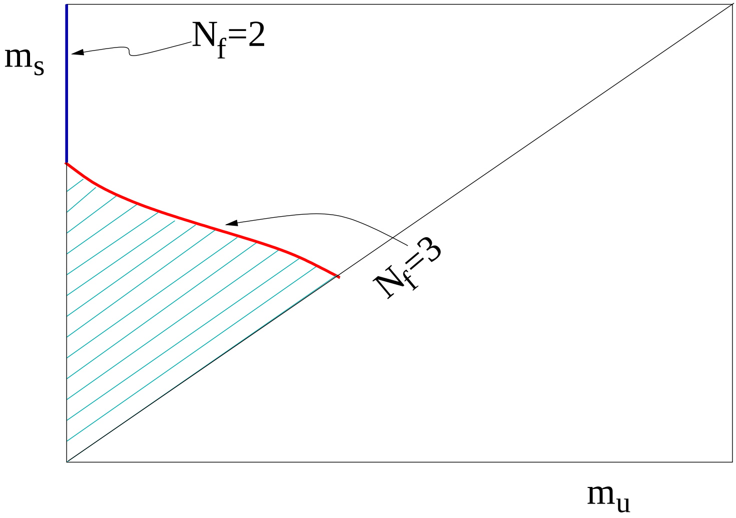

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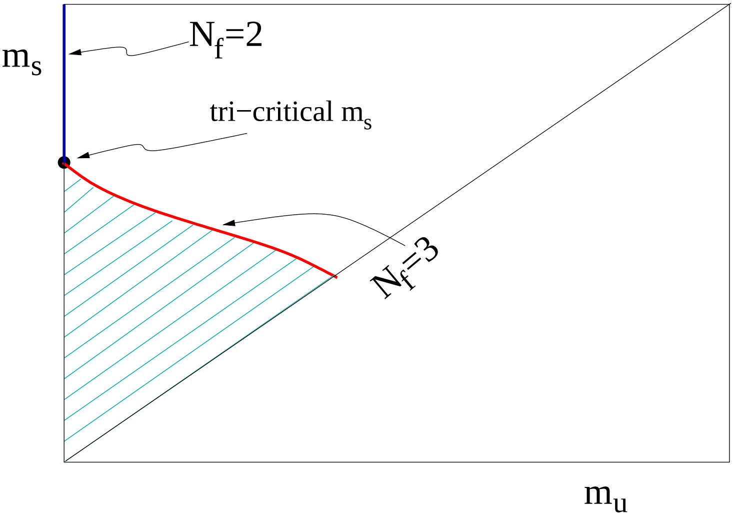

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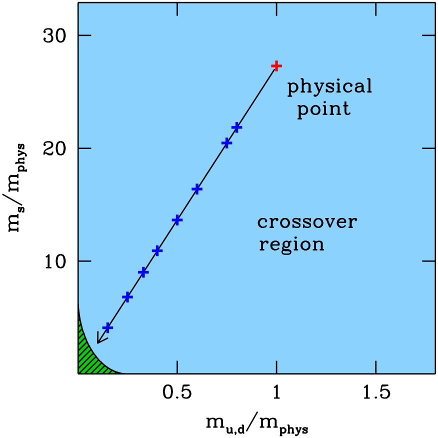
Lattice results for the Columbia Plot



In $N_f = 2 + 1$:

$$m_\pi^{crit} \begin{cases} = 0.07 m_\pi & (N_t = 4) \\ < 0.12 m_\pi & (N_t = 6) \end{cases}$$

Endrodi etal, 0710.0988 (2007)

Similarly for $N_f = 3$.

Karsch etal, hep-lat/0309121 (2004)

Lattice results for $N_f = 2 + 1$

1. Two independent lattice computations (now) agree on the position of the crossover temperature for physical quark mass ($m_\pi \simeq 140$ MeV):
$$T_c \simeq 170 \text{ MeV}.$$
Aoki etal, hep-lat/0611014 (2006); HotQCD, 2010.
2. Clear evidence that susceptibilities do not diverge: no critical point, definitely a cross over. BW: difference between "chiral" and "deconfinement" cross overs. HotQCD: no such difference.
3. Chiral ($m_\pi = 0$) critical point: not yet well determined. Current studies indicate that expected divergences do occur. However, the approach to infinities are not yet completely under control. Ejiri etal, 0909.5122 (2009)

Lattice results for $N_f = 1 + 1$

No significant change in T_c as $m_{\pi^0}/m_{\pi^\pm}$ is changed from 1 to 0.78 (physical value bracketed). Only one study; lattice spacings are coarse by today's standards; finite size scaling yet to be performed.

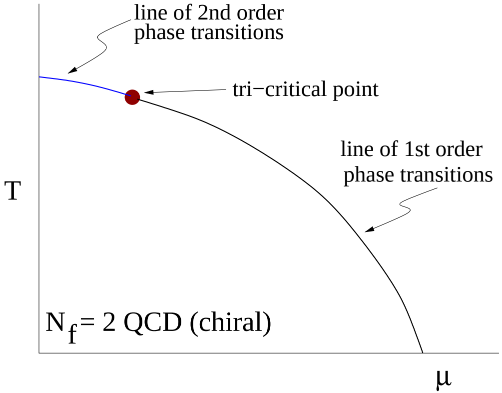
Gavai, SG, hep-lat/0208019 (2002)

$$\frac{T_c}{\Lambda_{\overline{MS}}} = \begin{cases} 0.49 \pm 0.02 & (m^2_{\pi^0}/m^2_{\pi^\pm} = 1) \\ 0.49 \pm 0.02 & (m^2_{\pi^0}/m^2_{\pi^\pm} = 0.78) \end{cases}$$

Both results extrapolated to the physical value of m_π/m_ρ.

Outline

The two flavour phase diagram



Rajagopal, Stephanov, Shuryak hep-ph/9806219 and hep-ph/9903292

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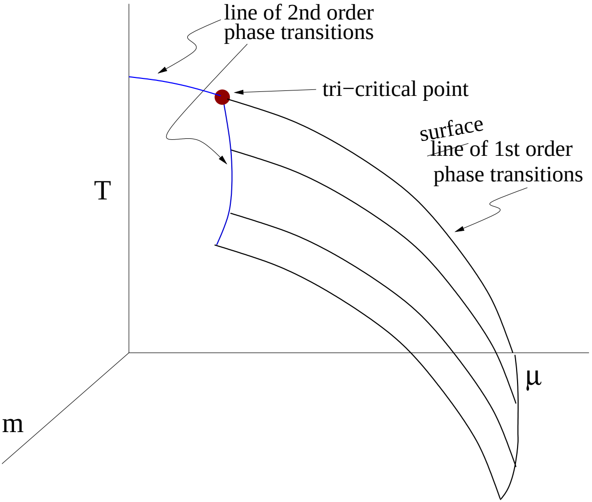

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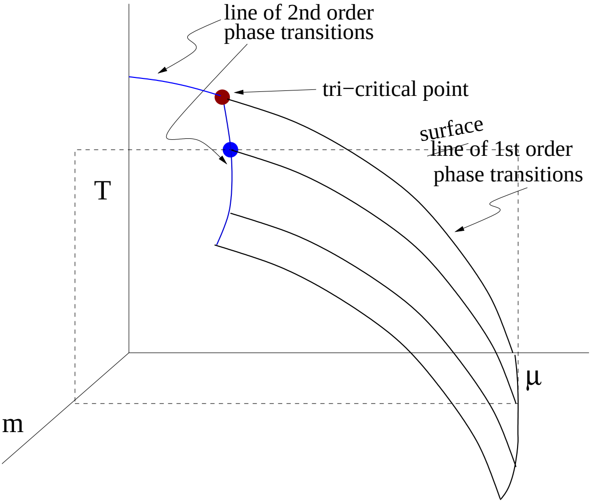

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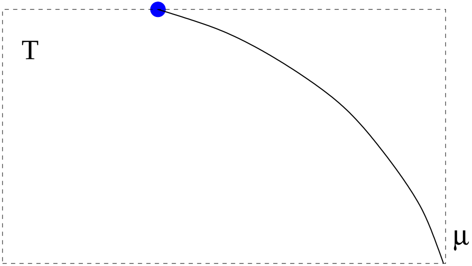

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Lattice setup

Lattice simulations impossible at finite baryon density: **sign problem**. Basic algorithmic problem in all Monte Carlo simulations: no solution yet.

Bypass the problem; make a Taylor expansion of the pressure:

$$P(T,\mu) = P(T) + \chi_B^{(2)}(T)\frac{\mu^2}{2!} + \chi_B^{(4)}(T)\frac{\mu^4}{4!} + \cdots$$

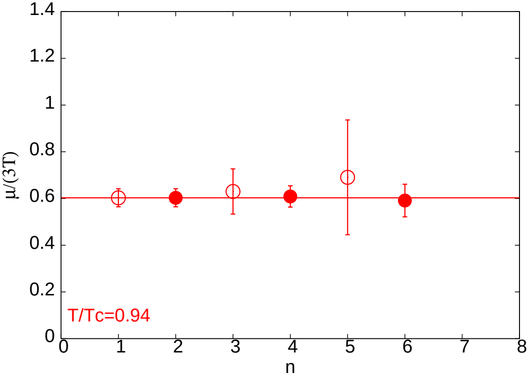
Series expansion coefficients evaluated at $\mu = 0$.
Implies

$$\chi_B^2(T,\mu) = \chi_B^{(2)}(T) + \chi_B^{(4)}(T)\frac{\mu^2}{2!} + \chi_B^{(6)}(T)\frac{\mu^4}{4!} + \cdots$$

Series fails to converge at the critical point.

, Gavai, SG, hep-lat/0303013 (2003)

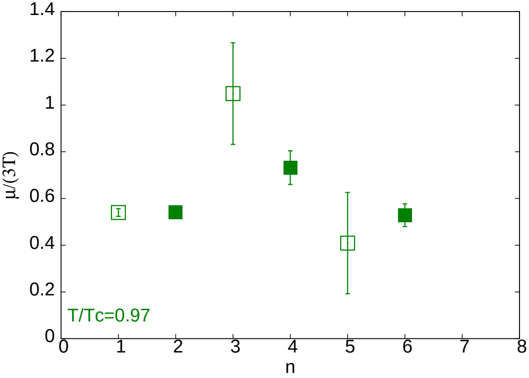
Series diverges



Radius of convergence of the series as a function of order ($a^{-1} = 1200$ MeV)

Gavai, SG, 0806.2233 (2008)

Series diverges



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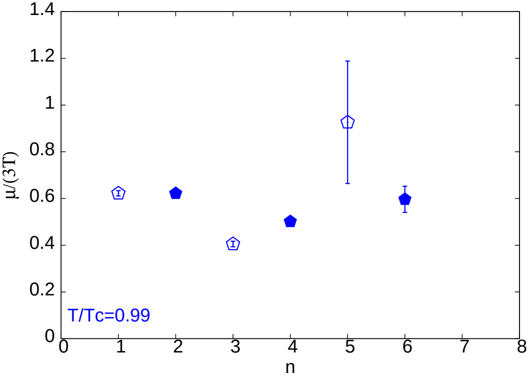

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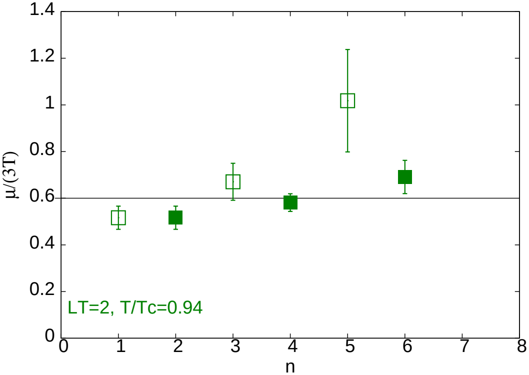

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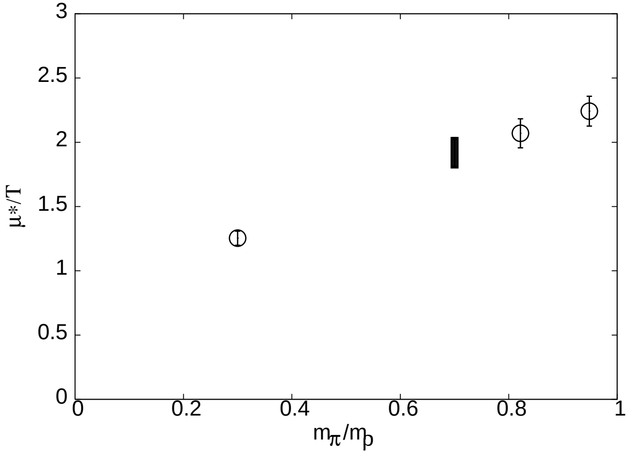
Dependence on quark mass



$a^{-1} = 800$ MeV

SG, hep-lat/0608022 (2006)

Systematic effects

1. Series expansion carried out to 8th order. What happens when order is increased? Intimately related to finite volume effects. Finite size scaling tested; works well

   Gavai, SG 2004, 2008

2. What happens when strange quark is unquenched (keeping the same action)? Numerical effects on ratios of susceptibility marginal when unquenching light quarks

   Gavai, SG, hep-lat/0510044; see also RBRC 2009; de Forcrand, Philipsen, 2007, 2009

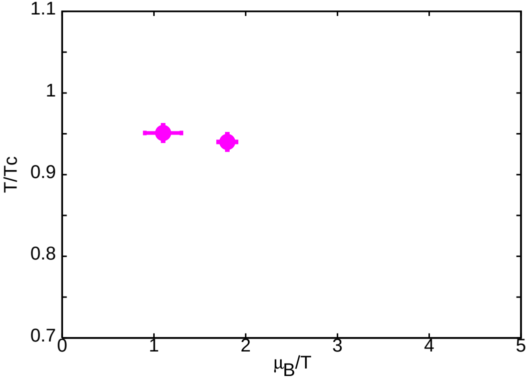
3. What happens when m_π is decreased? Estimate of μ_B^E may decrease somewhat: first estimates in

   Gavai, SG, Ray, nucl-th/0312010; see also Fodor, Katz 2001, 2002.

4. What happens in the continuum limit? Estimate of μ_B^E may increase somewhat

   Gavai, SG 2008; SG 2009

The critical point of QCD



$$\frac{\mu^E}{T^E} \simeq \begin{cases} 1.8 \pm 0.1 \quad N_f = 2,\ 1/a = 1200 \text{ MeV} & \text{Gavai, SG, 0806.2233 (2008)} \\ 1.5 \pm 0.4 \quad N_f = 2+1,\ 1/a = 800 \text{ MeV} & \text{BNL-Bielefeld-GSI, unpublished, 2010} \end{cases}$$

comparable m_π; normalized to same estimator.

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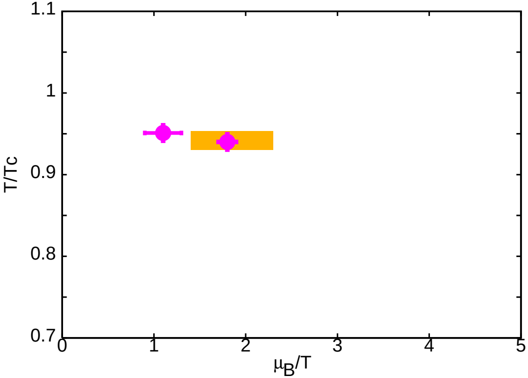

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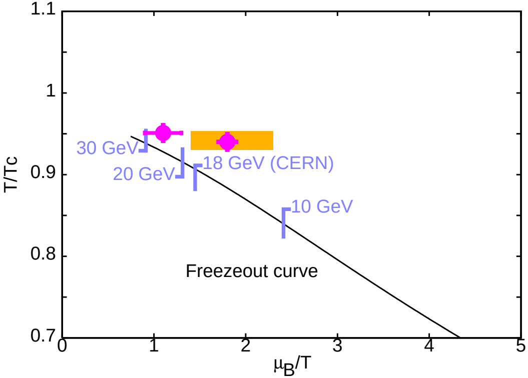

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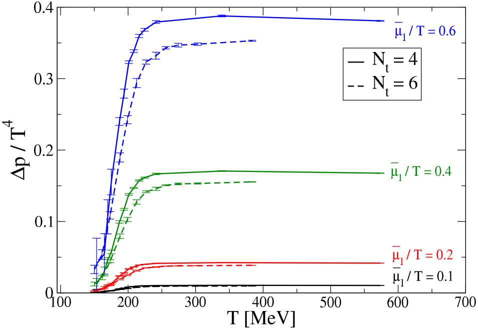
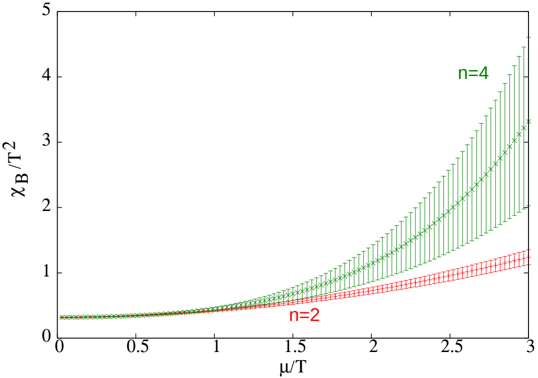
Extrapolating physical quantities



$$\Delta p = \chi_B^{(2)} \frac{\mu^2}{2!} + \chi_B^{(4)} \frac{\mu^4}{4!} + \cdots - \chi_S^{(2)} \frac{\mu_S^2}{2!} - \cdots$$

MILC Collaboration, 1003.5682 (2010)

Critical divergence: summation bad, resummation good



Infinite series diverges, but truncated series finite and smooth: sum is bad. Resummations needed to reproduce critical divergence. Padé resummation useful Gavai, SG, 0806.2233 (2008).

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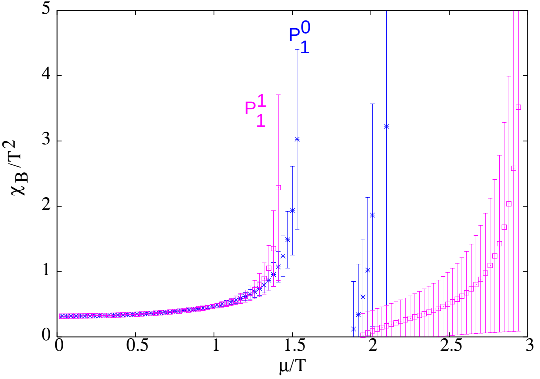

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Outline

Locating the critical end point in experiment

Measure the (divergent) width of momentum distributions

Stephanov, Rajagopal, Shuryak, hep-ph/9903292 (1999)

Better idea, use conserved charges, because at any normal (non-critical) point in the phase diagram:

$$P(\Delta B) = \exp\left(-\frac{(\Delta B)^2}{2VT\chi_B}\right). \qquad \Delta B = B - \langle B\rangle.$$

At any non-critical point the appropriate correlation length (ξ) is finite. If the number of independently fluctuating volumes ($N = V/\xi^3$) is large enough, then net B has Gaussian distribution: **central limit theorem**

Landau and Lifschitz

Bias-free measurement possible

Asakawa, Heinz, Muller, hep-ph/0003169 (2000); Jeon, Koch, hep-ph/0003168 (2000).

Is the top RHIC energy non-critical?

Check whether CLT holds.

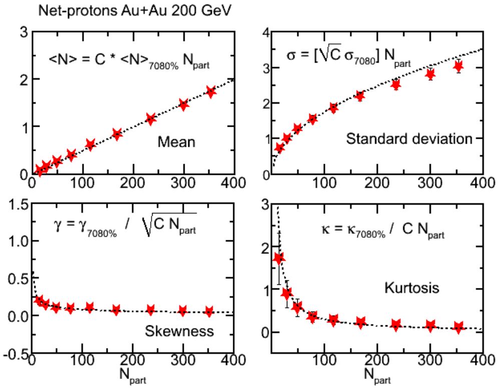
Recall the scalings of extensive quantity such as B and its variance σ^2, skewness, $\mathcal{S}$, and Kurtosis, $\mathcal{K}$, given by

$$B(V) \propto V, \quad \sigma^2(V) \propto V, \quad \mathcal{S}(V) \propto \frac{1}{\sqrt{V}}, \quad \mathcal{K}(V) \propto \frac{1}{V}.$$

Coefficients depend on T and μ. So make sure that the nature of the physical system does not change while changing the volume.

This is a check that microscopic physics is forgotten (except two particle correlations).

STAR measurements



Perfect CLT scaling: remember only $VT\chi_B$? or some other physics?

Can we recover microscopic physics?

Can we test QCD?

STAR Collaboration: QM 2009, Knoxville

What to compare with QCD

The cumulants of the distribution are related to Taylor coefficients—

$$[B^2] = T^3 V \left(\frac{\chi^{(2)}}{T^2} \right), \quad [B^3] = T^3 V \left(\frac{\chi^{(3)}}{T} \right), \quad [B^4] = T^3 V \chi^{(4)}.$$

T and V are unknown, so direct measurement of QNS not possible (yet). Define variance $\sigma^2 = [B^2]$, skew $\mathcal{S} = [B^3]/\sigma^3$ and Kurtosis, $\mathcal{K} = [B^4]/\sigma^4$. Construct the ratios

$$m_1 = \mathcal{S}\sigma = \frac{[B^3]}{[B^2]}, \qquad m_2 = \mathcal{K}\sigma^2 = \frac{[B^4]}{[B^2]}, \qquad m_3 = \frac{\mathcal{K}\sigma}{\mathcal{S}} = \frac{[B^4]}{[B^3]}.$$

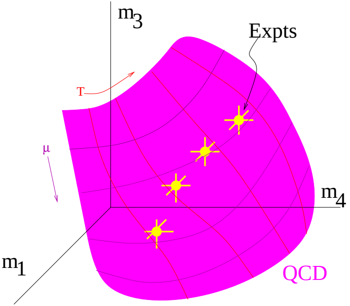
These are comparable with QCD provided all other fluctuations removed.

SG, 0909.4630 (2009)

How to compare with QCD

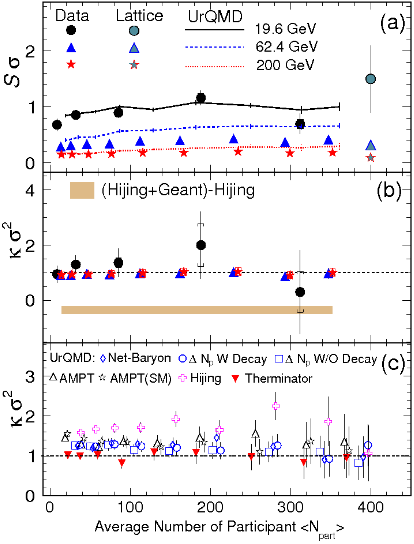
Since two parameters in QCD computations (T and μ), possible measurements lie on a surface. Make three independent measurements (also independent of fireball volume) to check this. Agreement of QCD and data implies control over other sources of fluctuations.

SG, 0909.4630 (2009)



If in equilibrium then CP implies non-monotonic behaviour of $m_1 = \mathcal{S}\sigma$, *etc.* with energy. Near CP system drops out of equilibrium: finite lifetime and finite size. Lack of agreement with QCD is signal of CP!

Microscopic non-Gaussianity



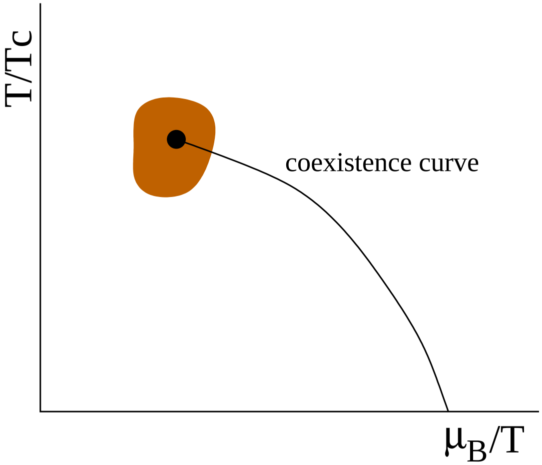
STAR Collaboration, 1004.4959 (2010)

Surprising agreement with lattice QCD: implies non-thermal sources of fluctuations are very small; temperature does not vary across the freezeout surface.

Gavai, SG, 1001.3796 (2010)

Lattice prediction: $\mathcal{K}\sigma^2 = 0.88 \pm 0.0$

How to find the critical point: be lucky!



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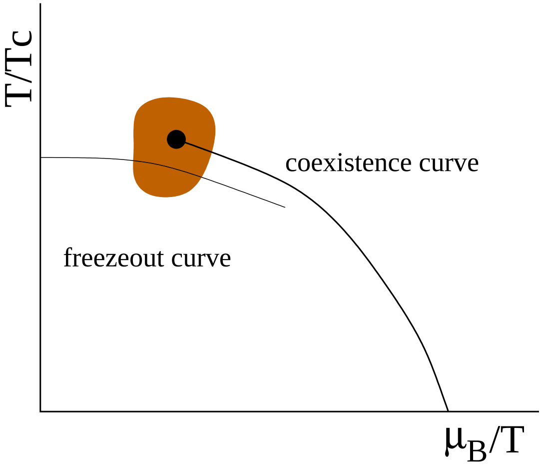

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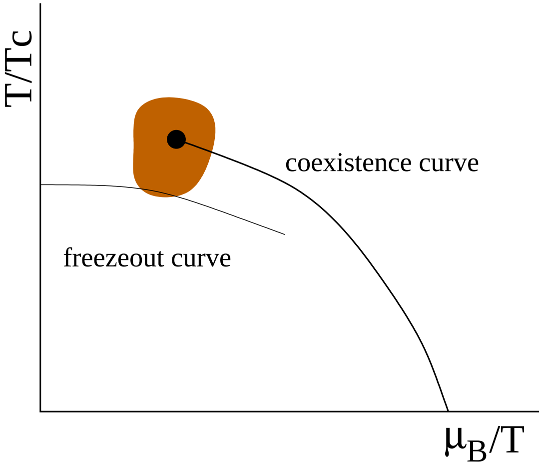

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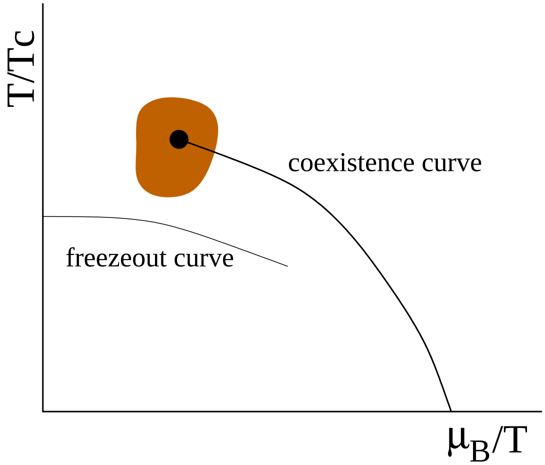
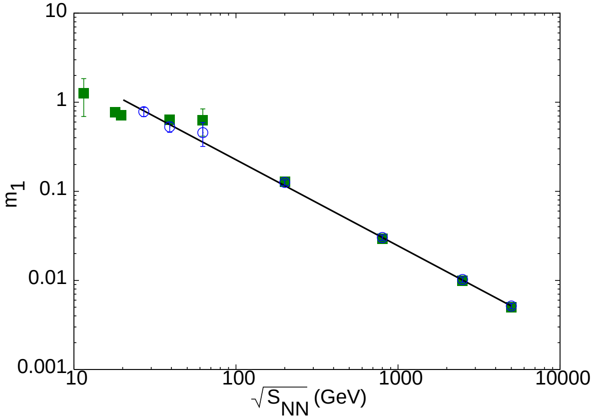


Lattice predictions along the freezeout curve



Filled boxes: $a = 1/(4T)$, unfilled circles: $a = 1/(6T)$.

Gavai, SG, 1001.3796 (2010)

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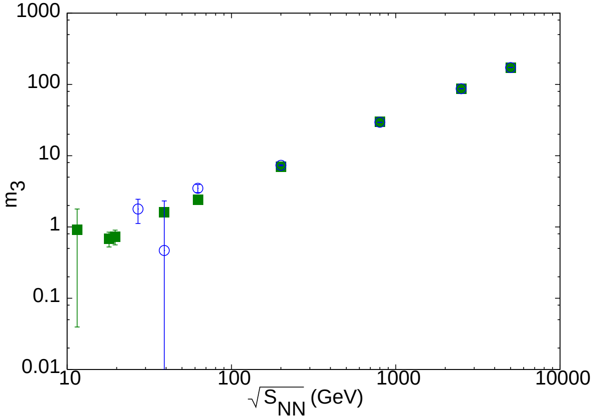

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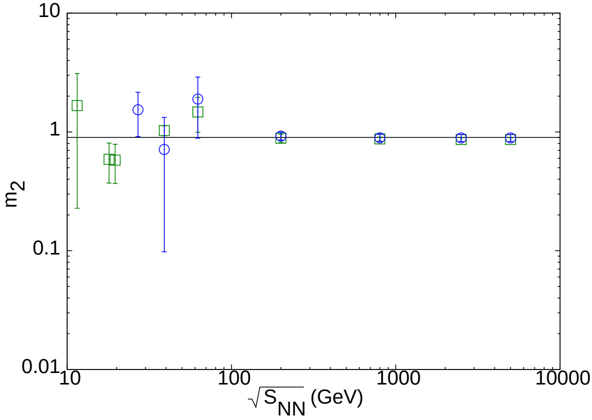

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Outline

Main results

- The strange quark is heavy; light quarks determine the shape of the phase diagram. The cross over temperature now under control: $T_c \simeq 170$ MeV. SU(2) flavour symmetry breaking unlikely to change T_c.
- Approach to zero quark mass is beginning to come under control for the first time. Consistency with continuum symmetry arguments may be established. (Computation very hard with small pion mass)
- Lattice determines series expansion of pressure; indicates a critical point in QCD. Range of predictions: $\mu^E/T^E \simeq 1.5$–2.5. Physical quantities can be found be resumming the series expansion (*e.g.*, Padé approximants).
- Extrapolation of lattice results to the experimentally known freezeout curve possible. First results in surprising agreement with experiment. Need to check CLT and determine ratios of moments.